Research Article

Application of a Multivariant Model in Determining the Contractual Benefit Obligation of a Defined Benefits Plan

Evaristo Diz Cruz¹, J. Tim Query²


²Ph.D., C.P.A., A.R.M. Professor & Mountain States Insurance Group Endowed Chair New Mexico State University, MSC 3FIN, P.O. Box 30001 Las Cruces, NM 88003 U.S.A. Ph. 575-646-5253 business.nmsu.edu/insurance tquery@nmsu.edu

Abstract: One of the most important aspects in actuarial valuations of a defined benefit plan type (severance schemes) is the forecast of the Projected Benefit Obligation (P) of a new individual or employee, given certain characteristics of a new employee. It is then a matter of estimating the actuarial liabilities of the company's employees in a sample and being able to estimate the new ones. This paper adds to previous research, using an application from company-level data, in the aggregate, where in this case they are employees of the same company.

Keywords: Social Welfare Benefits, Indemnifications under IAS 19, Regress Multivariate modeling R, Actuarial Statistics.

JEL: C 65, G 23, J64

I. Introduction & Objectives of Research

Pension funds are still a significant financial power in many countries. Given the wide diversity of funding levels currently maintained, there is obviously a lack of consensus on a suitable funding level. Research conducted by D’Arcy, et al. (1999) on public pension plans compares optimal funding levels based on a standard dependent on multi-factors the current pension obligation and growth rates of pension expenses and the tax base. Their study illustrates that funding levels should vary based on state-level economic conditions. In reality, however, they find states funding public pension plans at levels well below optimal values.

Other costs and risks faced by defined benefit plan providers include those arising from uncertain mortality, including longevity selection, mortality improvements, and unexpected systematic shocks. Using research based on longitudinal microdata on Australian pensioners, Fong, et al. (2015) quantifies the extent of longevity selection at both aggregate and scheme level. They also show that as the age-membership structure in a pension scheme matures, scheme-specific longevity selection risk and systematic shocks become quantitatively more important and have larger consequences for plan liabilities than aggregate selection risk or the impact of mortality improvements.

Notice that the proponents of the current model for market value of liability disclosures have latched onto private sector concepts of accounting and funding for identifying the benefits to value. However, these concepts do not value the contractual benefits earned to date under the voluntary exchange transaction, which occurs between employer and employee, and thus, should not be part of a fair value model. We should follow the labor economics principles more closely.

In our study we determine the actuarial liability of the projected obligation of a defined benefit plan type retroactive social benefits or "severance schemes." This is based on an actuarial valuation under IAS 19 made previously to all company employees at the individual level, where the prediction model Multivariate Linear Regression is obtained.

Accuracy with respect to pricing principles and financial engineering and pricing principles requires using the contractual obligation to determine what benefits to value. Care must be taken to not rely on other worthy goals exclusively, as we are pricing the fair value of the contractual benefit obligation here. That is the proper exit liability to value. That is the starting point for the process.
II. Description of Predictive Variables and Answer

The sample used to determine the PBO predictive model is characterized by the following variables:
1) Type of enterprise: Pharmaceutical Sector
2) Actuarial valuation: under IAS 19 to 12/31/2018
3) Interest rate: 4% real (i)
4) Retroactive Social Benefits (R)
5) Cumulative Social Benefits (G)
6) Social Benefits Differential (RG) =
7) Seniority in the company S
8) Integral reference salary ST
9) Age (and)
10) Gender / Sex (g)
11) ID
12) PBO calculated actuarial

II.a

II.b. Description of the Sample
1) 147 employees two distinct categories, consisting of 42 females and 105 males.
2) Classified by type of payroll: Employees 114 and workers 33
3) Years of average service: 4.4
4) Average age: 38.54
5) Social benefits payable: 480,599
6) Differential Benefits (retroactivity-guarantee): 76,973
7) Differential of the PBO (actuarial liability): 236.109
8) Benefit differential: 403,626
9) Real interest rate: 4%


a) Social benefits payable (PSP)

\[ PSP_t = \text{MAX}(R_t, G_t) \]

b) Retroactivity (R_t) \[ R_t = SI_t \times t \]

With a restriction on the time of service t that begins from the month of June of 1996 date of change of the labor law.

c) Guarantee or cumulative benefits \[ G_t = G_{T-1} + T_t SI_t \]
\( \hat{f}_t \): Aplicación rate between 2

\( SI_t \): Integral Salary in t

d) Performance Differential: \( \Delta_t = R_t - G_t \)
e) \( PBO_t \) differential actuarial liability

\( PBO_t = R_tA_t^s( \text{edad}) - G_tA_t^s( \text{edad}) \)

- \( s \): Rate of salary increase for \( R \), retroactivity
- \( s' \): Rate of salary increase for \( G \), the provision of guarantee
- \( A_t^s( \text{edad}) \): Contingent annuity evaluated at the rate \( s \) low risk of mortality and rotation.
- \( t \): nominal interest rate
- \( r \): real adjustment rate

\( (1 + i) = (1 + s)(1 + r) \) the equivalent nominal discount rate given a nominal wage increase rate.

\( \mu \): \( (1 + s) \)

\( \nu \): \( (1 + i)^{-1} \)

IV. Analysis of the Results

To process all the data, we used the statistical program R. This program is known to be very robust and has a large library of software. R provides a wide variety of statistical (linear and nonlinear modelling, classical statistical tests, time-series analysis, classification, clustering, …) and graphical techniques, and is highly extensible.

Next, each one of the statistical procedures used to find the model of best fit to the data is shown, respecting the principle of parsimony, a criterion for deciding among scientific theories or explanations. One should always choose the simplest explanation of a phenomenon, the one that requires the fewest leaps of logic.

IV.a. Descriptive Statistics of Each Variable

The following is the code in red and the results in blue, from the descriptive statistical analysis of the sample with R.

\texttt{> summary (pbo) }

LOT TT refers to the Organic labor law in Venezuela.

LOT TT SERVICES means accredited service (Seniority) under LOT TT for retroactivity calculation.

1) Gender Type of Payroll Service. X..RED.Servicio .x.LOTT .RED

<table>
<thead>
<tr>
<th></th>
<th>F: 42 EMPLOYEES: 114</th>
<th>M: 105 WORKERS: 33</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min.</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>1st Qtr</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Median</td>
<td>2,000</td>
<td>2,000</td>
</tr>
<tr>
<td>Mean</td>
<td>4.401</td>
<td>4.401</td>
</tr>
<tr>
<td>3rd Qtr</td>
<td>7.500</td>
<td>7.500</td>
</tr>
</tbody>
</table>

2) Age.x..RED .,GARANTIA.PS.x, PR.LOTT.T.x

<table>
<thead>
<tr>
<th></th>
<th>Min. : 17.00</th>
<th>Min. : 1383</th>
<th>Min.: 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Qtr:29.00</td>
<td>1st Qtr: 20233</td>
<td>1st Qtr: 24750</td>
<td></td>
</tr>
<tr>
<td>Median: 39.00</td>
<td>Median: 40519</td>
<td>Median: 103043</td>
<td></td>
</tr>
<tr>
<td>Mean: 38.54</td>
<td>Mean : 76973</td>
<td>Mean: 477089</td>
<td></td>
</tr>
<tr>
<td>3rd Qtr:46.00</td>
<td>3rd Qtr: 89639</td>
<td>3rd Qtr: 529750</td>
<td></td>
</tr>
<tr>
<td>Max: 72.00</td>
<td>Max: 686202</td>
<td>Max: 9271782</td>
<td></td>
</tr>
</tbody>
</table>

On average, there is a greater volume of social benefits due to retroactivity than to guarantee, as expected.

3) Differential of Social Indemnities to Pay Vs Guarantor

<table>
<thead>
<tr>
<th></th>
<th>Min: 1383</th>
<th>Min: 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Qtr: 28415</td>
<td>1st Qtr: 0</td>
<td></td>
</tr>
</tbody>
</table>
J. Tim Query et.al / Application of a Multivariant Model in Determining the Contractual Benefit Obligation of a Defined Benefits Plan

Median: 103043  Median: 43384
Mean: 480599   Mean: 403626
3rd Qtr: 529750 3rd Qtr: 400923
Max: 9271782  Max: 8585580

It is observed that the liability for the differential of benefits (PBO) is lower than the differential to be paid in nominal terms. This in principle varies depending on the supposed actuarial used and the densities of salaries and comprehensive service and / or seniority accumulated. In general, if the employee population is not very concentrated in high seniority with high salaries and the turnover is not very high, the $PBO \leq \Delta$ social benefits.

NULL PBO’S are obviously attributable to new employees.

4) CBO.DIF         Service Cost       Interest Cost
Min: 0            Min: 0            Min: 0
1st Qtr: 0        1st Qtr: 0        1st Qtr: 0
Median: 25667     Median: 88434     Median: 202255
Mean: 236109      Mean: 225055      Mean: 1860534
3rd Qtr: 235854   3rd Qtr: 325015   3rd Qtr: 1858522
Max: 5051093      Max: 2468197      Max: 39802469

V. Analysis of Years of Service Variability

> Boxplot (Servicio.x.LOTT..RED.-Sexo, data=PBO, id=list(method="y"))

The graph shows greater variability in the age of men compared to women. However, they exhibit an average of years of LOTTT service Similar.

Va. Variability of Guarantor Benefits

> Boxplot (GARANTIA.PS.x ~ Sex, data = PBO, id = list (method = "y"))
Similarly, the guarantee benefits of men are more volatile than that of women. This was to be expected given the volatility of the years of service of men. At the average level the guarantee of men is slightly higher.

VI. Conclusion

Prior to this paper we have previously investigated how to determine the actuarial liability or PBO Projected Benefit Obligation as a function of several explanatory variables, based on a sample of companies (Diz and Query, 2018).

In this additional contribution to the topic, the objective is basically the same, but the sample is constituted not by companies but by employees, all from the same company. The main finding is that in effect the PBO, can be expressed as a function of 2 or 3 predictor variables exhibiting a very simple linear structure, which facilitates the processing of additional data that would obviously be more cumbersome to calculate all the expected values of the contingent annuities that activate the binding formula of the Multivariant linear regression.

In the special case of this specific plan and subject to the previously defined assumptions and actuarial hypotheses on which the PBOs of the sample are based through a formal actuarial valuation, obtaining new PBO estimates from simple regressions is of great importance. computational value inclusive of cost to give global estimates in terms of quite reasonable orders of magnitude with very small forecast errors. The above undoubtedly represents a great advantage of fast and efficient calculation at low cost.

Of the two models studied, we adopt the simplest from the structural point of view and in line with the principle of parsimony.

In the annex you will find all the detailed calculations and the R code used to carry out all the estimates. The idea is to be able to reuse the same by changing the data to the actual rate used in the valuation.

A new analysis in line with this type of research would be to find an expression of the PBO based on different real interest rates, this would be very useful to know the impact of different cost scenarios by raising the interest rate.

The results of this research on the contrary are subject or conditioned.

\[
\Delta \text{PBO} = -74970.94 + 0.5544 \Delta P + 1788.19 \text{EDAD} + 4172.91 \text{SERVICIO LOT}
\]

Appendix Table of Contents - Statistical Computations

1. Matrix Analysis Correlatives of the variables
2. Distributions of Social Benefits to pay PSP
3. Distributions of Social Benefits PSP for Payroll
4. Comparison of PBO means by Sex and Payroll
5. Regression Model $PBO = F(\Delta PSP + SEX + EDAD)$
6. Diagnostic of the Regression Model
7. Confidence intervals at 95% of RegModel1 Regression Model Coefficients
8. Histogram of Residuals By Sex
9. Distribution of Residuals by Type of Payroll
10. Scatter graphics Plot of years of Service for Sex and for Retroactivity
11. Distribution by Sex and Payroll Type of Employees of the company in the sample.
12. Density of PBO By Sex
13. PBO Density by Payroll Type

APPENDICES

1. Correlative Matrix Analysis of Variables

```
>scatterplotMatrix(~Costo.Interés+Costo.Servicio+DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x+Edad.x..RED.+GARANTIA.PS.x+PBO.DIF
+ | Sexo,
+ regLine=FALSE, smooth=FALSE, diagonal=list(method="density"),
+ by.groups=TRUE, data=PBO)
```

From the matrix shown below you can see the densities of each variable and how the variables are correlated by sex. From the geometry thereof, both sexes are related in a similar manner without exhibiting large statistical differences.

Unlike the previous matrix where there were correlations of the variables by sex, quite similar, in the case of the matrix classified by type of number, different correlations between the payroll of employees and workers are observed.
The payroll of employees is much more dispersed than that of workers.

2. Distributions of Psp Social Benefits

```r
> with(PBO, Hist(PRESTACIONES.SOCIALES.A.PAGAR.EN.x, groups=Sexo, scale="frequency", breaks="Sturges", col="darkgray"))
```

3. Distributions of Social Benefits Psp Per Payroll

Greater density of PSP in men than in women and the results are repeated in the payroll of employees vs the payroll of.
4. Comparison of Cbo Stockings By Sex and Payroll

```r
> with (PBO, plotMeans(PBO.DIF, Sex, Type.Nomine, error.bars = "se", connect = TRUE, + legend.pos = "farright"))
```
5. Regression Model

> RegModel.1 <-
+ lm(PBO.DIF~DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x+Edad.x..RED.+Servicio.x.LOTT..RED.,
+ data=PBO)
> summary(RegModel.1)

Call:
lm(formula=PBO.DIF~DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x+EDAD.x..RED.+Servicio.x.LOTTT..RED.,data=PBO)
Residuals:
Min 1Q Median 3Q Max
-215176 -29776 -2518 22558 559515

Coefficients:

                         Estimate Std. Error t value
(Intercept)               -74970.946751  20143.068447   -3.722
DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x     0.554477      0.008294  66.852
Edad.x..RED.             1788.194041    553.86 3886   3.229
Servicio.x.LOTT..RED.     4172.911030   1793.696276   2.326
\[Pr(>|t|)\]

(Intercept)    0.000284 ***
DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x  < 2e-16 ***
Edad.x..RED.    0.001543 **
Servicio.x.LOTT..RED.   0.021400 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 70740 on 143 degrees of freedom
Multiple R-squared:  0.985, Adjusted R-squared:  0.9847
F-statistic:  3123 on 3 and 143 DF, p-value: < 2.2e-16

In total correspondence with the means by sex and type of payroll of the previous CBO graph.
All the previous analysis shows a good adjustment in terms of the $R^2$ with relatively small standard errors and some p-values that support the model in their sets. As can be seen in the following graphs on the diagnosis of the model, an almost normal behavior of the residuals is observed and no clearly differentiated patterns are seen that imply residual correlation.

6. Diagnosis of the Regression Model 1

From this analysis it can be deduced that the linear model is reasonably good to be able to make individual predictions of the PBO conditioned to the assumptions and hypotheses of the actuarial valuation of the sample.

```r
> oldpar <- pair ( oma = c ( 0 , 0 , 3 , 0 ) , mfrow = c ( 2 , 2 ) )
```

```r
> plot ( RegModel.1 )
```

7. Intervals of Confidence to 95% of the Coefficients of Regmodel 1 Regression Model

```r
> par(oldpar)
> library(MASS, pos=21)
> Confint(RegModel.1, level=0.95)
```

<table>
<thead>
<tr>
<th>Estimate</th>
<th>2.5</th>
<th>97.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>-74970.9467513</td>
<td>-114787.5929923</td>
</tr>
<tr>
<td>DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x</td>
<td>0.5544768</td>
<td>0.5380819</td>
</tr>
<tr>
<td>Edad.x..RED.</td>
<td>1788.1940409</td>
<td>693.3756184</td>
</tr>
<tr>
<td>Servicio.x.LOTTT..RED.</td>
<td>4172.9110302</td>
<td>627.3255951</td>
</tr>
<tr>
<td>DIFERENCIAL.A.PAGAR.vs.GARANTIA.EN.x</td>
<td>0.5708718</td>
<td></td>
</tr>
<tr>
<td>Edad.x..RED.</td>
<td>2883.0124633</td>
<td></td>
</tr>
<tr>
<td>Servicio.x.LOTTT..RED.</td>
<td>7718.4964653</td>
<td></td>
</tr>
</tbody>
</table>

The breadth of the 95% confidence intervals are quite reasonable, which guarantees good predictions.
Quality of Adjustment in Terms of A / CYB / C

```r
> AIC(RegModel.1)
[1] 3706.154
> BIC(RegModel.1)
[1] 3721.107
```

```r
> PBO <- within(PBO, {
+ fitted.RegModel.1 <- fitted(RegModel.1)
+ residuals.RegModel.1 <- residuals(RegModel.1)
+ rstudent.RegModel.1 <- rstudent(RegModel.1)
+ hatvalues.RegModel.1 <- hatvalues(RegModel.1)
+ cooks.distance.RegModel.1 <- cooks.distance(RegModel.1)
+ obsNumber <- 1:nrow(PBO)
+ })
```

8. Histogram of Residuals by Sex

```r
> with(PBO, Hist(residuals.RegModel.1, groups=Sexo, scale="frequency",
+ breaks="Sturges", col="darkgray"))
```

9. Distribution of Residuals by Type of Payroll

```r
> with(PBO, Hist(residuals.RegModel.1, groups=Tipo.de.Nomina, scale="frequency",
+ breaks="Sturges", col="darkgray"))
```
10. Graphics Scatter Plot of Years of Service by Sex for Retroactivity

It is observed that in the case of men there is a greater density of employees with services over 14 years, which implies a greater differential liability.

As in the previous graph, it is evident that the liability for the type of payroll is much higher in men, especially with services over 14 years.
11. Distribution by Sex and Type of Payroll of Employees of the Company in the Sample

```r
> with(PBO, Barplot(Sexo, by=Tipo.de.Nomina, style="divided", legend.pos="above", + xlab="Sexo", ylab="Frequency"))
```

```r
> densityPlot(PBO.DIF~Sexo, data=PBO, bw=bw.SJ, adjust=1, kernel=dnorm, + method="adaptive")
```

12. Pbo Density by Sex
13. Pbo Density by Type of Payroll

```r
> densityPlot(PBO.DIF~Tipo.de.Nomina, data=PBO, bw=bw.SJ, adjust=1, kernel=dnorm,
+    method="adaptive")
```

BIBLIOGRAPHY

D’Arcy, Stephen, James H. Dulebohn, and Pyungsuk Oh (1999). Optimal Funding of State Employee...

Diaz de Rada, V. (1999). Data analysis techniques for social researchers: practical applications with SSPS for Windows. Madrid: Ra-Ma


JRI Vol 82, Issue 1 pp. 33-64 March 2015


Valero, F.J. Options in financial instruments, Ariel, Barcelona, 1988

